UNITARY MINIMAL CONFORMAL WEIGHTS OF DILUTE A_L LATTICE MODELS

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Abstract

A fusion hierarchy of functional equations with an $su(3)$ structure is solved for the central charges and conformal weights of the dilute A_L lattice models in branch 2 to obtain the unitary minimal values $c = 1 - \frac{6}{L(L+1)}$ and $\Delta_{t,s} = \frac{[(L+1)t-Ls]^2-1}{4L(L+1)}$ with $s = 1, 2, \ldots, L$ and $t = 1, 2, \ldots, L-1$.

The dilute A–D–E lattice models [1] are of considerable interest, firstly, because they are solvable in the presence of a symmetry breaking field and, secondly, because they provide lattice realizations [2] of the complete A–D–E classification [3] of modular invariant partition functions of unitary minimal conformal field theories with $c < 1$. Here we indicate briefly how the central charges and conformal weights of the dilute A_L lattice models can be calculated restricting our attention to branch 2.

The face weights of the dilute A_L models at criticality are

$$
\begin{aligned}
W\left(\begin{matrix} d & c \\ a & b \end{matrix}\,\middle|\, u\right) &= \rho_1(u)\delta_{a,b,c,d} + \rho_2(u)\delta_{a,b,c}A_{a,d} + \rho_3(u)\delta_{a,c,d}A_{a,b} \\
&+ \sqrt{\frac{S_a}{S_b}}\rho_4(u)\delta_{b,c,d}A_{a,b} + \sqrt{\frac{S_c}{S_a}}\rho_5(u)\delta_{a,b,d}A_{a,c} + \rho_6(u)\delta_{a,b}\delta_{c,d}A_{a,c} \\
&+ \rho_7(u)\delta_{a,d}\delta_{c,b}A_{a,b} + \rho_8(u)\delta_{a,c}A_{a,b}A_{a,d} + \sqrt{\frac{S_aS_c}{S_bS_d}}\rho_9(u)\delta_{b,d}A_{a,b}A_{b,c}.
\end{aligned} \tag{1}
$$

Here the heights a, b, c, d take the values $1, 2, \ldots, L$, the adjacency matrix is given by $A_{a,b} = \delta_{a,b-1} + \delta_{a,b+1}$, the generalized Kronecker delta $\delta_{a,b,c,\ldots} = 1$ when $a = b = c = \ldots$ and 0 otherwise, and $\rho_j(u)$ with $j = 1, 2, \ldots, 9$ are certain trigonometric functions in the spectral parameter u. The crossing factors S_a are the nonnegative elements of the Perron-Frobenius eigenvector of the adjacency matrix and, in branch 2, $0 < u < 3\lambda$ where the crossing parameter is $\lambda = \frac{\pi}{4}\frac{L+2}{L+1}$.

The commuting row transfer matrices of the dilute lattice models satisfy [4] functional equations in the form of a fusion hierarchy with an $su(3)$ structure

$$\boldsymbol{T}_0^{(n,0)}\boldsymbol{T}_n^{(1,0)} = \boldsymbol{T}_0^{(n-1,1)} + \boldsymbol{T}_0^{(n+1,0)} \tag{2}$$

$$\boldsymbol{T}_1^{(0,m)}\boldsymbol{T}_0^{(0,1)} = \boldsymbol{T}_0^{(0,m+1)} + f_0\boldsymbol{T}_1^{(1,m-1)} \tag{3}$$

$$\boldsymbol{T}_0^{(n,0)}\boldsymbol{T}_n^{(0,m)} = \boldsymbol{T}_0^{(n,m)} + f_{n-1}\boldsymbol{T}_0^{(n-1,0)}\boldsymbol{T}_{n+1}^{(0,m-1)} \tag{4}$$

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where $\boldsymbol{T}_k^{(0,0)} = \mathbf{I}$, $\boldsymbol{T}_k^{(1,0)} = \boldsymbol{T}(u+2k\lambda)$, $\boldsymbol{T}_0^{(0,1)} = (-1)^N \frac{s(u+2\lambda)s(u-3\lambda)}{s(2\lambda)s(3\lambda)} \boldsymbol{T}_{1/2}^{(1,0)}$ and $f_k = f(u+2k\lambda)$ with

$$f(u) = (-1)^N \frac{s(u-3\lambda)s(u-2\lambda)s(u-\lambda)s(u+2\lambda)s(u+3\lambda)s(u+4\lambda)}{s(2\lambda)^3 s(3\lambda)^3} \tag{5}$$

and $s(u) = \sin^N u$. Here $\boldsymbol{T}_k^{(n,m)} = \boldsymbol{T}_{(1,0)}^{(n,m)}(u+2k\lambda)$ is the row transfer matrix of the fused model of fusion type $(1,0)$ in the horizontal direction and (n,m) in the vertical direction where the fusion levels are labelled by representations (n,m) of $su(3)$. The fusion hierarchy is compatible with the $su(3)$ fusion rules

$$\begin{aligned} (n,m)\otimes(1,0) &= (n+1,m)\oplus(n-1,m+1)\oplus(n,m-1) \\ (n,m)\otimes(0,1) &= (n,m+1)\oplus(n+1,m-1)\oplus(n-1,m) \end{aligned} \tag{6}$$

and closes on the $su(3)$ weight lattice with

$$\boldsymbol{T}_0^{(n,m)} = 0 \qquad \text{if } n+m \geq 2L. \tag{7}$$

If we now define

$$\boldsymbol{t}_0^q = \frac{\boldsymbol{T}_0^{(q+1,0)}\boldsymbol{T}_1^{(q-1,0)}}{\boldsymbol{T}_0^{(0,q)}} \tag{8}$$

we can obtain [5] the TBA equations

$$\frac{\boldsymbol{t}_0^q \boldsymbol{t}_1^q}{\boldsymbol{t}_{1/2}^q} = \frac{[\mathbf{I}+\boldsymbol{t}_0^{q+1}][\mathbf{I}+\boldsymbol{t}_1^{q-1}]}{[\mathbf{I}+\boldsymbol{t}_{1/2}^q]}. \tag{9}$$

These equations can be solved [5] for the central charges and conformal weights of the dilute A_L models, as given in the abstract, following the methods of Klümper and Pearce [6]. Interestingly, although the answers are the same as for the $su(2)$ case [6], the calculation now uses an $su(3)$ dilogarithm identity [7] instead of an $su(2)$ dilogarithm identity.

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