

Surface Free Energies and Surface Critical Behaviour of the ABF Models with Fixed Boundaries¹

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Abstract

In a previous paper, we introduced reflection equations for interaction-round-a-face (IRF) models and used these to construct commuting double-row transfer matrices for solvable lattice spin models with fixed boundary conditions. In particular, for the Andrews-Baxter-Forrester (ABF) models, we derived special functional equations satisfied by the eigenvalues of the commuting double-row transfer matrices. Here we introduce a generalized inversion relation method to solve these functional equations for the surface free energies. Although the surface free energies depend on the boundary spins we find that the associated surface critical exponent $\alpha_s = (7 - L)/4$ is independent of the choice of boundary.

1 Introduction

We have recently demonstrated how, by generalizing the work of Sklyanin [1], fixed boundary conditions may be imposed upon interaction-round-a-face (IRF) models whilst preserving solvability [2]. Specifically, we have shown that the Yang-Baxter equations and the boundary reflection equations imply commutativity of double-row transfer matrices. Related results have also been obtained by the authors of [3] and [4].

Furthermore, we have found solutions to the reflection equations for the Andrews-Baxter-Forrester (ABF) models [5]. Consideration of the fusion [6, 7, 8, 9] of this

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model with fixed boundaries leads to functional relations for the double-row transfer matrices, and hence their eigenvalues. In this paper we solve these functional equations in the thermodynamic limit to obtain the surface free energy, from which we determine the surface critical exponent α_s .

In the remainder of this section we summarize the results of [2] which are needed for our calculation. For further details we refer the reader to this reference. In Section 2 we demonstrate the solution of the inversion relation for the bulk free energy, and then generalize this method to the surface free energy.

1.1 The ABF models with fixed boundaries

The ABF models [5] are restricted solid-on-solid models in which sites on the lattice take values in the set $\{1, 2, 3, \dots, L\}$ subject to the condition that the values of sites adjacent on the lattice must differ by ± 1 . The Boltzmann weights depend on a *crossing parameter* $\lambda = \pi/(L + 1)$, and a *spectral parameter* u . In the regimes of interest we have $0 < u < \lambda$. The non-zero face weights are given by

$$W \begin{pmatrix} a \pm 1 & a \\ a & a \mp 1 \end{pmatrix} = \frac{\vartheta_1(\lambda - u)}{\vartheta_1(\lambda)} \quad (1.1)$$

$$W \begin{pmatrix} a & a \pm 1 \\ a \mp 1 & a \end{pmatrix} = \left(\frac{\vartheta_1((a-1)\lambda)\vartheta_1((a+1)\lambda)}{\vartheta_1^2(a\lambda)} \right)^{1/2} \frac{\vartheta_1(u)}{\vartheta_1(\lambda)} \quad (1.2)$$

$$W \begin{pmatrix} a & a \pm 1 \\ a \pm 1 & a \end{pmatrix} = \frac{\vartheta_1(a\lambda \pm u)}{\vartheta_1(a\lambda)}. \quad (1.3)$$

The $\vartheta_1(u) = \vartheta_1(u, p)$ are elliptic theta functions with nome p . We define $p = \exp(-\varepsilon)$ with $\varepsilon > 0$, corresponding to regime III. The critical limit is $p \rightarrow 0$. The boundary weights depend on an additional arbitrary complex parameter ξ , which may be different for the left and right boundaries. The non-zero boundary weights are

$$K \begin{pmatrix} a \pm 1 & a \\ a & a \end{pmatrix} = \left(\frac{\vartheta_1((a \pm 1)\lambda)}{\vartheta_1(a\lambda)} \right)^{1/2} \frac{\vartheta_1(u \pm \xi)\vartheta_1(u \mp a\lambda \mp \xi)}{\vartheta_1^2(\lambda)}. \quad (1.4)$$

There is another form of the boundary weights which, at criticality, is independent of u and ξ

$$K \begin{pmatrix} a \pm 1 & a \\ a & a \end{pmatrix} = \left(\frac{\vartheta_1((a \pm 1)\lambda)}{\vartheta_1(a\lambda)} \right)^{1/2} \frac{\vartheta_4(u \pm \xi)\vartheta_4(u \mp a\lambda \mp \xi)}{\vartheta_4^2(\lambda)}. \quad (1.5)$$

This expression is obtained from (1.4) simply by making a complex shift in ξ . However, in this paper we will take ξ to be real and consider only the two forms of the boundary weights (1.4) and (1.5). From the face weights and boundary weights we construct a double-row transfer matrix $\mathbf{D}(u)$. For a lattice of width N , the entry

of the transfer matrix corresponding to the rows of spins $\mathbf{a} = \{a_1, \dots, a_{N+1}\}$ and $\mathbf{b} = \{b_1, \dots, b_{N+1}\}$ is defined diagrammatically by

$$\langle \mathbf{a} | \mathbf{D}(u) | \mathbf{b} \rangle =$$

The solid spins $\{c_1, \dots, c_{N+1}\}$ are summed over. As the boundary weights are diagonal, we must have $a_1 = b_1$ and $a_{N+1} = b_{N+1}$. Furthermore, these boundary spins, which we will call a_L and a_R , are fixed to the same values for all entries in the transfer matrix. The parameters ξ_L and ξ_R are similarly fixed for all entries. Defined in this way, the double-row transfer matrix exhibits the crossing symmetry

$$\mathbf{D}(\lambda - u) = \mathbf{D}(u). \quad (1.6)$$

More importantly, however, the double-row transfer matrices form a commuting family,

$$\mathbf{D}(u)\mathbf{D}(v) = \mathbf{D}(v)\mathbf{D}(u). \quad (1.7)$$

This implies that the eigenvectors of $\mathbf{D}(u)$ are independent of u , so that functional equations satisfied by the transfer matrix are also satisfied by its eigenvalues. In particular, all eigenvalues of the transfer matrix satisfy the crossing symmetry (1.6). It should be emphasized that all the matrices in a commuting family share the same boundary spins a_L and a_R , and the same values of ξ_L and ξ_R .

The values $\xi = \pm\lambda/2$ deserve special mention, since for these choices the isotropic lattice, $u = \lambda/2$, has all boundary spins fixed. This is easily seen from the definition (1.4), as, for fixed a , only one of the choices $a \pm 1$ gives a non-zero boundary weight. The non-zero boundary weights then contribute only a constant factor to each entry the transfer matrix. Aside from this trivial factor, the lattice exhibits pure fixed boundary conditions, with boundary spins alternating $\{a, a + 1, a, a + 1, \dots\}$ or $\{a, a - 1, a, a - 1, \dots\}$.

Just as in the case of periodic boundary conditions, the face weights and boundary weights may be fused [6, 7, 8, 9] to form new solvable models with fixed boundary conditions. The functional equations which result have the same form as in the periodic case, with the addition of some order 1 factors related to the boundary.

2 Functional equations

It has been shown in [2] that the eigenvalues of the ABF models with fixed boundary conditions at fusion level $1 \times q$ satisfy the inversion identity hierarchy

$$s_{q-1}s_{q+1}D^q(u)D^q(u+\lambda) = \gamma_{q-1}^q s_{-1}s_{2q+1}f_{-1}f_q + s_q^2 D^{q-1}(u+\lambda)D^{q+1}(u), \quad (2.1)$$

where $1 \leq q \leq L-1$, and

$$s_k = \vartheta_1(2u + (k-1)\lambda), \quad \gamma_k^r = \alpha_k^r \beta_k^r, \quad f_k = (-1)^N \left[\frac{\vartheta_1(u+k\lambda)}{\vartheta_1(\lambda)} \right]^{2N}. \quad (2.2)$$

In terms of the function

$$\theta_k^r(u) = \prod_{j=0}^{r-1} \frac{\vartheta_1(u + (k-j)\lambda)}{\vartheta_1(\lambda)}, \quad (2.3)$$

we can write α_k^r and β_k^r as

$$\begin{aligned} \alpha_k^r(u) &= \theta_k^r(u - \xi_L) \theta_k^r(u + \xi_L) \theta_k^r(u - \xi_R) \theta_k^r(u + \xi_R) \\ \beta_k^r(u) &= \theta_{k-a_L}^r(u - \xi_L) \theta_{k+a_L}^r(u + \xi_L) \theta_{k-a_R}^r(u - \xi_R) \theta_{k+a_R}^r(u + \xi_R). \end{aligned} \quad (2.4)$$

If the second form of the boundary weights (1.5) is used, the ϑ_1 functions in (2.3) should be changed to ϑ_4 functions. The closure of the inversion identity hierarchy is governed by the conditions

$$D^{-1}(u) = D^L(u) = 0, \quad D^0(u) = f_{-1}, \quad D^{L-1}(u) = (-1)^N f_{-2} \alpha_{L-2}^L / \beta_{-2}^1. \quad (2.5)$$

If we write the eigenvalues $D^q(u)$ with their bulk and surface terms separated

$$D^q(u) \sim D_b^q(u) D_s^q(u) \quad \text{as } N \rightarrow \infty, \quad (2.6)$$

then, by virtue of the inversion relation for the fused face weights [6], $D_b^q(u)$ satisfies the functional relation

$$D_b^q(u) D_b^q(u+\lambda) = f_{-1} f_q, \quad (2.7)$$

2.1 The bulk free energy

Equation (2.7) implies that, between the inversion points $u = (1-q)\lambda/2$ and $(3-q)\lambda/2$, the bulk partition function per site satisfies the functional equation

$$\kappa_b^q(u) \kappa_b^q(u+\lambda) = \frac{\vartheta_1(\lambda-u) \vartheta_1(u+q\lambda)}{\vartheta_1(\lambda)^2}. \quad (2.8)$$

This is the same inversion relation as in the case of periodic conditions, which is to be expected since the boundary conditions should not affect the bulk behaviour. Equation (2.8) has been solved previously [8, 9, 10], but we include the solution here for completeness, and as an introduction to the generalized methods that follow. We use the standard techniques developed by Baxter [11]. The assumption that the solutions are analytic between the inversion points, and that they may be analytically continued a small distance outside the strip, along with the relation (2.8) and the crossing symmetry

$$D^q((2-q)\lambda - u) = D^q(u) \quad (2.9)$$

uniquely determines the free energies. The assumption of analyticity may be justified by studying the zeros of the largest eigenvalue $D^q(u)$ for large finite N . In the critical case, it is seen that for the fusion level $1 \times q$, the strip $-q\lambda/2 < \text{Re}(u) < (4-q)\lambda/2$ is free of order N zeros [12]. Furthermore, there can be no poles inside this strip, since for finite N the independence of the eigenvectors on u implies that the eigenvalues are simply linear combinations of products of Boltzmann weights. Since none of the Boltzmann weights have poles, neither can the eigenvalues. This assumption of analyticity implies that the logarithms of the partition functions may be expanded in a Laurent series in powers of $\exp(2\pi u/\varepsilon)$,

$$\ln \kappa_b^q(u) = \sum_{k=-\infty}^{\infty} c_k e^{2k\pi u/\varepsilon}. \quad (2.10)$$

We rewrite the right hand side of (2.8) using the ‘‘conjugate modulus’’ transformation, which, in terms of the function

$$E(x, p) = \prod_{n=1}^{\infty} (1 - p^{n-1}x)(1 - p^n x^{-1})(1 - p^n), \quad (2.11)$$

is given by

$$\vartheta_1(u, p) = \left(\frac{\pi}{\varepsilon}\right)^{1/2} e^{-(u-\pi/2)^2/\varepsilon} E(e^{-2\pi u/\varepsilon}, \tilde{q}^2) \quad (2.12)$$

$$\vartheta_4(u, p) = \left(\frac{\pi}{\varepsilon}\right)^{1/2} e^{-(u-\pi/2)^2/\varepsilon} E(-e^{-2\pi u/\varepsilon}, \tilde{q}^2) \quad (2.13)$$

where $p = \exp(-\varepsilon)$ and $\tilde{q} = \exp(-\pi^2/\varepsilon)$ are conjugate nomes. With both sides of (2.8) expanded in powers of $\exp(2\pi u/\varepsilon)$, we match coefficients and impose the crossing symmetry (2.9) to obtain the solution

$$\begin{aligned} \ln \kappa_b^q(u) = c_0(u) &+ \sum_{k=1}^{\infty} \frac{\cosh[(\pi - 2\lambda)\pi k/\varepsilon]}{k \sinh(\pi^2 k/\varepsilon)} \\ &- \sum_{k=1}^{\infty} \frac{\cosh[(\pi - (q+1)\lambda)\pi k/\varepsilon] \cosh[((2-q)\lambda - 2u)\pi k/\varepsilon]}{k \sinh(\pi^2 k/\varepsilon) \cosh(\lambda\pi k/\varepsilon)}, \end{aligned} \quad (2.14)$$

where

$$c_0(u) = \frac{1}{2\varepsilon} [(\pi - q\lambda)(q-1)\lambda + 2u((2-q)\lambda - u)]. \quad (2.15)$$

The analytic continuation of this function to the region $-q\lambda/2 < \text{Re}(u) < (4-q)\lambda/2$ gives the bulk behaviour of the largest eigenvalue of the transfer matrix

$$D_b^q(u) \sim [\kappa_b^q(u)]^{2N} \quad \text{as } N \rightarrow \infty. \quad (2.16)$$

From (2.14) it is easy to show that inside the interval $-q\lambda/2 < u < (2-q)\lambda/2$,

$$\kappa_b^{q-1}(u+\lambda)\kappa_b^{q+1}(u) = \kappa_b^q(u)\kappa_b^q(u+\lambda) \quad \text{when } q > 1. \quad (2.17)$$

When $q = 1$, it can also be shown that inside $-\lambda/2 < u < \lambda/2$,

$$\ln \left| \frac{\kappa_b^{q-1}(u+\lambda)\kappa_b^{q+1}(u)}{\kappa_b^q(u)\kappa_b^q(u+\lambda)} \right| = -\frac{\pi}{2\varepsilon}(\lambda - 2|u|) - \sum_{k=1}^{\infty} \frac{\sinh(\lambda - 2|u|)\pi k/\varepsilon}{k \cosh(\lambda\pi k/\varepsilon)}, \quad (2.18)$$

so that

$$|\kappa_b^{q-1}(u+\lambda)\kappa_b^{q+1}(u)| < |\kappa_b^q(u)\kappa_b^q(u+\lambda)| \quad \text{when } q = 1. \quad (2.19)$$

Putting together equations (2.8), (2.16), (2.17) and (2.19), we therefore obtain

$$\lim_{N \rightarrow \infty} \left(\frac{D_b^{q-1}(u+\lambda)D_b^{q+1}(u)}{f_{-1}(u)f_q(u)} \right) = \begin{cases} 0 & \text{when } q = 1, \\ 1 & \text{when } q > 1, \end{cases} \quad (2.20)$$

which is consistent with the critical bulk behaviour described in [12]. We need this result for the derivation that follows.

2.2 The surface free energy

Recalling the separation of the bulk and surface terms (2.6), and using the functional equation (2.7) and the bulk behaviour of the eigenvalues (2.20), the inversion identity hierarchy in the thermodynamic limit becomes, for $-q\lambda/2 < \text{Re}(u) < (2-q)\lambda/2$,

$$s_{q-1}s_{q+1}D_s^q(u)D_s^q(u+\lambda) = \begin{cases} \gamma_0^1 s_{-1}s_3 & \text{if } q = 1, \\ \gamma_{q-1}^q s_{-1}s_{2q+1} + s_q^2 D_s^{q-1}(u+\lambda)D_s^{q+1}(u) & \text{otherwise.} \end{cases} \quad (2.21)$$

In the case of the unfused model, $q = 1$, we therefore have an inversion relation which allows us to calculate the surface free energy⁵. Explicitly,

$$\kappa_s(u)\kappa_s(u+\lambda) = \frac{\vartheta_1(2\lambda - 2u)\vartheta_1(2\lambda + 2u)}{\vartheta_1(\lambda - 2u)\vartheta_1(\lambda + 2u)} \epsilon^L(u)\epsilon^R(u), \quad (2.22)$$

⁵In the general case of $p \times q$ fusion, an analogous functional equation is derived when $q = p$.

where $\epsilon^L(u)$ and $\epsilon^R(u)$ are defined by, for a and ξ corresponding to the left or right boundary as appropriate,

$$\epsilon^{L,R}(u) = \theta_0^1(u - \xi)\theta_0^1(u + \xi)\theta_{-a}^1(u - \xi)\theta_a^1(u + \xi). \quad (2.23)$$

In the unfused case, the crossing symmetry of the eigenvalues implies that

$$\kappa_s(\lambda - u) = \kappa_s(u). \quad (2.24)$$

The solution of the inversion relation for the surface free energy proceeds in a similar fashion as that for the bulk free energy, although we must justify the assumption of analyticity separately. There can of course be no poles between the inversion points for the same reason as there are none in the bulk. However, we are now concerned about order 1 zeros inside $0 < \text{Re}(u) < \lambda$. Certainly zeros occur on the line $\text{Re}(u) = \lambda/2$, but, guided by the derivation of conformal weights for periodic boundary conditions [12], we associate these zeros with finite-size (order $1/N$) corrections rather than surface effects.

Numerical studies show that zeros do occur on the real u axis inside the strip $-\lambda/2 < \text{Re}(u) < 3\lambda/2$, but never inside the strip $0 < \text{Re}(u) < \lambda$. For certain values of ξ , zeros occur at the inversion points $u = 0$ and λ . These values are determined by the zeros of $\epsilon^{L,R}(u)$, and are found to be $\xi = 0$, $-a\lambda$ and $(L + 1 - a)\lambda$. For all other values of ξ (up to periodicity), the interval between the inversion points is free of zeros. We therefore conclude that the surface partition function per site, $\kappa_s(u)$, is analytic in this region.

With this assumption of analyticity, and the imposition of the crossing symmetry, the solution of (2.22) is

$$\begin{aligned} \ln \kappa_s(u) = & 2 \sum_{k=1}^{\infty} \frac{\sinh[(\pi - 3\lambda)\pi k/\varepsilon] \sinh(\lambda\pi k/\varepsilon) \cosh[2(\lambda - 2u)\pi k/\varepsilon]}{k \sinh(\pi^2 k/\varepsilon) \cosh(2\lambda\pi k/\varepsilon)} \\ & + (\pi - 3\lambda)\lambda/\varepsilon + \ln \kappa_s^L(u) + \ln \kappa_s^R(u), \end{aligned} \quad (2.25)$$

where $\ln \kappa_s^L$ and $\ln \kappa_s^R$ are given for generic a and ξ by

$$\begin{aligned} \ln \kappa_s^{L,R}(u) = & c_0(u) + 2 \sum_{k=1}^{\infty} \frac{\cosh[(\pi - 2\lambda)\pi k/\varepsilon]}{k \sinh(\pi^2 k/\varepsilon)} \\ & - 2 \sum_{k=1}^{\infty} \frac{\cosh[(a\lambda + \xi - |\xi|)\pi k/\varepsilon] \cosh[(\pi - a\lambda - \xi - |\xi|)\pi k/\varepsilon] \cosh[(\lambda - 2u)\pi k/\varepsilon]}{k \sinh(\pi^2 k/\varepsilon) \cosh(\lambda\pi k/\varepsilon)} \end{aligned} \quad (2.26)$$

and

$$c_0(u) = \frac{1}{\varepsilon} \left[(a\lambda + \xi)(\pi - 2\xi) + (|\xi| - 2\lambda)\pi + (2 - a^2)\lambda^2 + 2u(\lambda - u) \right]. \quad (2.27)$$

In deriving this expression, ξ is assumed to satisfy the inequality

$$-\pi < (1 - a)\lambda < \xi < (L - a)\lambda < \pi. \quad (2.28)$$

We note that, as one would expect, the height reversal transformation $a \rightarrow L + 1 - a$ and $\xi \rightarrow -\xi$ leaves (2.26) unchanged. If the second form of the boundary weights (1.5) is used, each term in the sums of (2.26) should be multiplied by $(-1)^k$, which alters the critical behaviour.

The temperature variable t is identified in [5] to be given by $t = p^2$. The behaviour in the critical limit $t \rightarrow 0^+$ is found by applying the Poisson summation formula to the above expression for $\ln \kappa_s$ (2.25). We find that the leading-order singularities of $\ln \kappa_s$ have the form

$$f_{\text{sing}} \sim \begin{cases} t^{\pi/4\lambda} & \text{if } L \equiv 0 \text{ or } 1 \pmod{4}, \\ t^{\pi/4\lambda} \ln t & \text{if } L \equiv 3 \pmod{4}. \end{cases} \quad (2.29)$$

When $L \equiv 2 \pmod{4}$, $\ln \kappa_s$ is regular. The leading singularity of $\ln \kappa_s^{L,R}$ is in general of higher order than that of $\ln \kappa_s$. In addition, $\ln \kappa_s^{L,R}$ is regular in the following situations:

$$L \text{ even } \begin{cases} \xi = k\lambda, & k \in \mathbb{Z} \\ \xi > 0 \text{ and } a \text{ odd} \\ \xi < 0 \text{ and } a \text{ even} \end{cases} \quad L \text{ odd } \begin{cases} \xi = (2k + 1)\lambda/2, & k \in \mathbb{Z} \\ a \text{ odd} \end{cases}$$

These exceptions aside, $\ln \kappa_s^{L,R}$ has the leading-order singularities

$$f_{\text{sing}}^{L,R} \sim \begin{cases} t^{\pi/2\lambda} & \text{if } L \text{ is even,} \\ t^{\pi/2\lambda} \ln t & \text{if } L \text{ is odd.} \end{cases} \quad (2.30)$$

When the second form of the boundary weights (1.5) is used, the leading-order singularities are, aside once again from the above exceptions,

$$f_{\text{sing}}^{L,R} \sim \begin{cases} t^{\pi/4\lambda} & \text{if } L \text{ is even,} \\ t^{\pi/4\lambda} \ln t & \text{if } L \text{ is odd.} \end{cases} \quad (2.31)$$

In this case the function $\ln \kappa_s^{L,R}$ vanishes at criticality.

The surface critical exponent α_s is defined in analogy with the bulk critical exponent α [13, 14],

$$C_b \sim |t|^{-\alpha} \quad C_s \sim |t|^{-\alpha_s}, \quad (2.32)$$

with the specific heats

$$C_b = \frac{\partial^2 f_b}{\partial t^2} \quad C_s = \frac{\partial^2 f_s}{\partial t^2}. \quad (2.33)$$

Our result for the surface critical exponent α_s is therefore

$$\alpha_s = \frac{7 - L}{4}. \quad (2.34)$$

The ABF model with $L = 3$ corresponds to the two-dimensional Ising model. In this case we have

$$f_{\text{sing}} \sim t \ln t, \quad (2.35)$$

in agreement with the calculations of McCoy and Wu [15] for free boundary conditions, but with a lattice rotated by 45° with respect to the one considered by them.

3 Conclusion

From the inversion identity hierarchy, and from the known solution of the bulk free energy, we have derived an inversion relation for the surface free energy of the ABF models with fixed boundary conditions. We have solved this inversion relation, subject to justifiable analyticity assumptions, and thus obtained an expression for the surface free energy. Finally, we have analysed the critical behaviour of the surface free energy to obtain the surface critical exponent α_s .

In this paper we have considered the bulk form of the free energy and its surface (order 1) correction. In a future publication we will study the finite-size (order $1/N$) corrections at criticality, and hence derive the central charges and conformal weights of the ABF models with fixed boundaries.

After this work was completed, we received the preprints [16] and [17], in which the authors derive surface critical exponents of the eight-vertex and ABF models using methods similar to ours.

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References

- [1] E. K. Sklyanin, J. Phys. A **21** (1988) 2375.
- [2] R. E. Behrend, P. A. Pearce and D. L. O'Brien, *Interaction-round-a-face models with fixed boundary conditions: the ABF fusion hierarchy*, University of Melbourne Preprint, hep-th/9507118, to be published in J. Stat. Phys.
- [3] P. P. Kulish, *Yang-Baxter equation and reflection equations in integrable models*, hep-th/9507070.
- [4] C. Ahn and W. M. Koo, *Boundary Yang-Baxter equation in the RSOS representation*, Preprint EWHA-TH, SNUTP-95-080, hep-th/9508080.
- [5] G. E. Andrews, R. J. Baxter and P. J. Forrester, J. Stat. Phys. **35** (1984) 193.
- [6] E. Date, M. Jimbo, T. Miwa and M. Okado, Lett. Math. Phys. **12** (1986) 209.
- [7] E. Date, M. Jimbo, T. Miwa and M. Okado, Lett. Math. Phys. **14** (1987) 97.
- [8] E. Date, M. Jimbo, A. Kuniba, T. Miwa and M. Okado, Nucl. Phys. B **290** [FS20] (1987) 231.
- [9] E. Date, M. Jimbo, A. Kuniba, T. Miwa and M. Okado, Adv. Stud. Pure Math. **16** (1988) 17.
- [10] V. V. Bazhanov and N. Yu. Reshetikhin, Int. J. Mod. Phys. A **4** (1989) 115.
- [11] R. J. Baxter, J. Stat. Phys. **28** (1982) 1.
- [12] A. Klümper and P. A. Pearce, Physica A **183** (1992) 304.
- [13] K. Binder, *in* "Phase Transitions and Critical Phenomena", Volume 8, (C. Domb and J. L. Lebowitz, eds). Academic Press, London, 1983.
- [14] H. W. Diehl, *in* "Phase Transitions and Critical Phenomena", Volume 10, (C. Domb and J. L. Lebowitz, eds). Academic Press, London, 1986.
- [15] B. M. McCoy and T. T. Wu, "The Two-Dimensional Ising Model". Harvard University Press, Cambridge, Massachusetts, 1973.
- [16] M. T. Batchelor and Y. K. Zhou, *Surface Critical Phenomena and Scaling in the Eight-Vertex Model*, ANU preprint MRR 069-95, cond-mat/9510152.
- [17] Y. K. Zhou and M. T. Batchelor, *Surface Critical Phenomena in Interaction-Round-a-Face Models*, ANU preprint MRR-070-95, cond-mat/9511008.